Preference Determinants of Inequality Reducing Income Taxation

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Abstract

The link between income inequality and progressive taxation has long been considered a fundamental normative foundation for income tax progressivity. This paper furnishes necessary and sufficient conditions on primitives under which various subclasses of progressive taxes are inequality reducing. The results are a strict generalization of those in Carbonell-Nicolau and Llavador (2018), and confer a degree of useful flexibility on the theory, in that it allows the analyst to expand the universe of consumer preferences by suitably restricting the set of marginal-rate progressive taxes. As an illustration of the result's practical implications, we provide a precise characterization of the subclass of (progressive) taxes that are inequality reducing for the CES and the quasi-linear utility functions. These families of preferences are pervasive in surveys and textbooks on labor supply and fiscal policy.

Keywords: progressive taxation; income inequality; incentive effects of taxation.
JEL classifications: D63, D71.

1 Introduction

The link between income inequality and progressive taxation uncovered in the seminal works of Jakobsson (1976) and Fellman (1976) has long been considered a fundamental normative foundation for income tax progressivity.[1] In a recent paper, Carbonell-Nicolau and Llavador

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[1]The literature on the redistributive effects of tax systems was initiated by Musgrave and Thin (1948). The contributions of Jakobsson (1976) and Fellman (1976) led to a large body of literature on the foundations of tax progressivity (see, e.g., Kakwani (1977); Hemming and Keen (1983); Eichhorn et al. (1984); Liu (1985); Formby et al. (1986); Thon (1987); Latham (1988); Thistle (1988); Moyes (1988, 1994); Le Breton et al. (1996); Ebert and Moyes (2000); Ju and Moreno-Ternero (2008)).

(2018) extended the classic result of Jakobsson (1976) and Fellman (1976)—according to which average-rate progressive, and only average-rate progressive income taxes, reduce income inequality—to the case of endogenous income. There it was shown that marginal-rate progressivity—in the sense of increasing marginal tax rates on income—is a necessary condition for tax structures to be inequality reducing, and necessary and sufficient conditions on preferences were identified under which progressive and only progressive taxes are inequality reducing. While this result circumvents the difficulties and the negative results emphasized by other authors in their attempts to incorporate the disincentive effects of taxation (see, e.g., Allingham (1979) and Ebert and Moyes (2003, 2007)), it confines attention to the conditions under which the set of *all* marginal-rate progressive taxes are inequality reducing. Evidently, requiring *larger* families of tax schedules to be inequality reducing results in *stronger* conditions on consumer preferences. In fact, the conditions derived in Carbonell-Nicolau and Llavador (2018) may be regarded, in some cases, as overly restrictive: while they are fulfilled by some standard classes of preferences—such as the Cobb-Douglas preferences and the so-called *GHH preferences* (see Greenwood et al., 1988)—this paper illustrates that there are important preference classes—such as the CES and the quasi-linear families of utility functions—that violate them. A natural question, therefore, is whether there are *subclasses* of marginal-rate progressive tax schedules that are inequality reducing for *larger* collections of preferences.This paper identifies necessary and sufficient conditions on consumer preferences ensuring that various subclasses of progressive taxes are inequality reducing.

We consider continuous, piecewise linear, nondecreasing tax schedules that preserve the ranking of pre-tax incomes. The allowable constraints on taxes take the form of subsidies (negative taxes) and/or subsets of $[0\%, 100\%)$ for the marginal tax rates. Each lower bound on the subsidy received by the poorest individual, together with a subset of possible marginal tax rates for each tax bracket, gives rise to a subclass of marginal-rate progressive tax schedules. The main result of the paper (Theorem 4) characterizes, for each such subclass $\mathscr{T}$, the family of preferences that renders the members of $\mathscr{T}$ inequality reducing.

The result obtained here is a strict generalization of that in Carbonell-Nicolau and Llavador (2018), and confers a degree of useful flexibility on the theory, in that it allows the analyst to expand the universe of consumer preferences by suitably restricting the set of marginal-rate progressive taxes. As an illustration of the result's practical implications, we provide a precise characterization of the subclass of (progressive) taxes that are inequality reducing for the CES and the quasi-linear utility functions. These preferences are pervasive in surveys and textbooks on labor supply and fiscal policy (see, e.g., Pencavel, 1986; Killingsworth and Heckman, 1986; Auerbach and Kotlikoff, 1987; Keane, 2011; Blundell et al., 2016). In addition, the CES utility function (often in its Cobb-Douglas form) is widely used in the literature on life-cycle models (see, e.g., Heckman and MaCurdy, 1982; French, 2005; Blundell et al., 2016), while static models with fixed costs traditionally work with quasi-linear preferences (Cogan, 1981).

Both for the CES and the quasi-linear utility functions, we find that a large enough subsidy suffices for a tax schedule to be inequality reducing if and only if the elasticity of substitution between leisure and consumption is sufficiently large (see Proposition 1 and Proposition 2 in Section 4 and the intuition developed in Subsections 4.1 and 4.2). Thus, the elasticity of substitution between leisure and consumption proves to be an important determinant of the distributional effects of progressive income taxation.

The remainder of the paper is organized as follows. Section 2 introduces the formal setting. It defines the set of piecewise linear tax schedules, it describes the agent's problem and introduces Lorenz dominance as the inequality criterion. The main results are presented in Section 3. Section 4 studies applications to the CES and the quasi-linear utility functions, providing a precise characterization of the inequality-reducing subclasses of progressive taxes for these preferences, as well as the intuition behind this characterization. All proofs are relegated to the Appendix.

2 Preliminaries

The setting is the same as that of Carbonell-Nicolau and Llavador (2018). There are n individuals. The utility function is given by a continuous utility function $u : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ defined over consumption-labor pairs $(c,l) \in \mathbb{R}_+ \times [0,1]$ such that $u(\cdot,l)$ is strictly increasing in c for each $l \in [0,1)$, and $u(c,\cdot)$ is strictly decreasing in l for each $c > 0$. The map u is assumed strictly quasiconcave on $\mathbb{R}_{++} \times [0,1)$ and twice continuously differentiable on $\mathbb{R}_{++} \times (0,1)$. For $(c,l) \in \mathbb{R}_{++} \times (0,1)$, let

$$MRS(c,l) := -\frac{u_l(c,l)}{u_c(c,l)}$$

denote the marginal rate of substitution of labor for consumption, where

$$u_c(c,l) := \frac{\partial u(c,l)}{\partial c} \quad \text{and} \quad u_l(c,l) := \frac{\partial u(c,l)}{\partial l}.$$

We assume that for each $c > 0$,

$$\lim_{l \to 1^-} MRS(c,l) = +\infty \quad \text{and} \quad \lim_{l \to 0^+} MRS(c,l) < +\infty. \tag{1}$$

The set of all utility functions satisfying the above conditions is denoted by $\mathscr{U}$.

We restrict attention to nondecreasing and order-preserving piecewise linear tax schedules.

Definition 1. Let $(\alpha_0, \boldsymbol{t}, \overline{\boldsymbol{y}}) = \big(\alpha_0, (t_0, \ldots, t_K), (\overline{y}_0, \ldots, \overline{y}_K)\big)$, where $\alpha_0 \geq 0$, $K \in \mathbb{Z}_+$, $t_k \in [0,1)$ for each $k \in \{0,\ldots,K\}$, $t_k \neq t_{k+1}$ whenever $k \in \{0,\ldots,K-1\}$ and $K \geq 1$, and $0 = \overline{y}_0 < \cdots < \overline{y}_K$. A $(K+1)$***-bracket piecewise linear tax schedule*** is a real-valued map T on $\mathbb{R}_+$ uniquely determined by $(\alpha_0, \boldsymbol{t}, \overline{\boldsymbol{y}})$ as follows:

$$T(y) := \begin{cases} -\alpha_0 + t_0 y & \text{if } 0 = \overline{y}_0 \le y \le \overline{y}_1, \\ -\alpha_0 + t_0\overline{y}_1 + t_1(y - \overline{y}_1) & \text{if } \overline{y}_1 < y \le \overline{y}_2, \\ \quad\vdots & \quad\vdots \\ -\alpha_0 + t_0\overline{y}_1 + t_1(\overline{y}_2 - \overline{y}_1) + \cdots + t_{K-1}(\overline{y}_K - \overline{y}_{K-1}) + t_K(y - \overline{y}_K) & \text{if } \overline{y}_K < y. \end{cases}$$

Here $T(y)$ is interpreted as the tax liability for gross income level y. We write $(\alpha_0, \boldsymbol{t}, \overline{\boldsymbol{y}})$ and the associated map T interchangeably. Note that for $K = 0$, $(\alpha_0, t_0, \overline{y}_0 = 0)$ is a linear tax with intercept α_0 and marginal tax rate t_0; for $K = 1$, $(\alpha_0, (t_0, t_1), (\overline{y}_0, \overline{y}_1))$ is a two-bracket tax with intercept α_0, marginal tax rates t_0 and t_1, and bracket threshold $\overline{y}_1$; and so on.

The set of piecewise linear tax schedules is denoted by $\mathscr{T}$.

The following notion of tax progressivity, which requires that marginal tax rates be nondecreasing with income, plays an essential role in our results.

Definition 2. A tax schedule $T \in \mathscr{T}$ is ***marginal-rate progressive*** if it is a convex function.

The set of all marginal-rate progressive tax schedules in $\mathscr{T}$ is denoted by $\mathscr{T}_{prog}$.

Linear tax schedules play an important role in the analysis, and it is convenient to introduce their formal definition.

Definition 3. A tax schedule $T \in \mathscr{T}$ is ***linear*** if $T(y) = -b + ty$ for all $y \in \mathbb{R}_+$ and some $b \ge 0$ and $t \in [0,1)$.

Denote the set of all linear tax schedules in $\mathscr{T}$ as $\mathscr{T}_{lin}$.

Individuals differ in their abilities. An ***ability distribution*** is a vector $\boldsymbol{a} = (a_1, \ldots, a_n) \in \mathbb{R}^n_{++}$ such that $a_1 \le \cdots \le a_n$. The set of all ability distributions is denoted by $\mathscr{A}$.

An agent of ability $a > 0$ who chooses $l \in [0,1]$ units of labor and faces a tax schedule $T \in \mathscr{T}$ consumes $c = al - T(al)$ units of the good and obtains a utility of $u(c,l)$. Thus, the agent's problem is

$$\max_{l \in [0,1]} u(al - T(al), l). \tag{2}$$

Because the members of $\mathscr{U}$ and $\mathscr{T}$ are continuous, for given $u \in \mathscr{U}$, $a > 0$, and $T \in \mathscr{T}$, the optimization problem in (2) has a solution, although it need not be unique. A ***solution function*** is a map $l^u : \mathbb{R}_{++} \times \mathscr{T} \to [0,1]$ such that $l^u(a,T)$ is a solution to (2) for each $(a,T) \in \mathbb{R}_{++} \times \mathscr{T}$. The ***pre-tax*** and ***post-tax income functions*** associated to a solution function l^u, denoted by $y^u : \mathbb{R}_{++} \times \mathscr{T} \to \mathbb{R}_+$ and $x^u : \mathbb{R}_{++} \times \mathscr{T} \to \mathbb{R}_+$ respectively, are given by

$$y^u(a,T) := al^u(a,T) \quad \text{and} \quad x^u(a,T) := al^u(a,T) - T\left(al^u(a,T)\right).$$

Given $a > 0$, let $U^a : \mathbb{R}_+ \times [0,a] \to \mathbb{R}$ be defined by $U^a(c,y) := u(c, y/a)$. For $(c,y,a) \in \mathbb{R}^3_{++}$ with $y < a$, define

$$U^a_c(c,y) := \frac{\partial U^a(c,y)}{\partial c}, \quad U^a_y(c,y) := \frac{\partial U^a(c,y)}{\partial y}, \quad \text{and } \eta^a(c,y) := -\frac{U^a_y(c,y)}{U^a_c(c,y)}.$$

The following condition was introduced by Mirrlees (1971, Assumption B, p. 182) and termed *agent monotonicity* by Seade (1982).[2]

Definition 4. A utility function $u \in \mathscr{U}$ satisfies ***agent monotonicity*** if $\eta^a(c,y) \geq \eta^{a'}(c,y)$ for each $(c,y) \in \mathbb{R}^2_+$ and $0 < a < a'$ with $y < a$.

The set of all the members of $\mathscr{U}$ satisfying agent monotonicity is denoted by $\mathscr{U}^*$.

Inequality comparisons are based on the standard relative Lorenz ordering. An ***income distribution*** is a vector $\boldsymbol{z} = (z_1, \dots, z_n) \in \mathbb{R}^n_+$ of incomes arranged in increasing order, i.e., $z_1 \leq \cdots \leq z_n$. Given two income distributions $\boldsymbol{z} = (z_1, \dots, z_n)$ and $\boldsymbol{z}' = (z'_1, \dots, z'_n)$ with $z_n, z'_n > 0$, we say that $\boldsymbol{z}$ is at least as equal as $\boldsymbol{z}'$ if ***$\boldsymbol{z}$ Lorenz dominates $\boldsymbol{z}'$***, i.e., if

$$\frac{\sum_{i=1}^k z_i}{\sum_{i=1}^n z_i} \geq \frac{\sum_{i=1}^k z'_i}{\sum_{i=1}^n z'_i}, \quad \text{for all } k \in \{1, \dots, n\}.$$

For $u \in \mathscr{U}^*$, and given pre-tax and post-tax income functions y^u and x^u, an ability distribution $\boldsymbol{a} = (a_1, \dots, a_n) \in \mathscr{A}$ and a tax schedule $T \in \mathscr{T}$ determine a ***pre-tax income distribution***

$$\boldsymbol{y}^u(\boldsymbol{a}, T) := \left(y^u(a_1, T), \dots, y^u(a_n, T)\right)$$

and a ***post-tax income distribution***

$$\boldsymbol{x}^u(\boldsymbol{a}, T) := \left(x^u(a_1, T), \dots, x^u(a_n, T)\right).$$[3]

In the absence of taxation, i.e., if $T \equiv 0$, one has $\boldsymbol{y}^u(\boldsymbol{a}, 0) = \boldsymbol{x}^u(\boldsymbol{a}, T)$.

The following is the central notion of inequality reducing tax schedule.

Definition 5. Let $u \in \mathscr{U}$. A tax schedule $T \in \mathscr{T}$ is ***income inequality reducing with respect to*** u, which we denote as u-***iir***, if $\boldsymbol{x}^u(\boldsymbol{a}, T)$ Lorenz dominates $\boldsymbol{y}^u(\boldsymbol{a}, 0)$ for each ability distribution $\boldsymbol{a} := (a_1, \dots, a_n) \in \mathscr{A}$ and for each pre-tax and post-tax income functions y^u and x^u.

Observe that the '*iir*' relation compares post-tax income distributions with the income distribution in the absence of taxation, and requires the former to be at least as equal as the latter, according to the relative Lorenz criterion.

[2]This is a mild requirement: any preference violating the agent monotonicity condition would necessarily treat consumption as an inferior good (Myles, 1995, p. 136).

[3]Under the agent monotonicity condition, in both cases the vector components are arranged in increasing order. See Carbonell-Nicolau and Llavador (2018).

3 The results

To begin, we recapture a result from Carbonell-Nicolau and Llavador (2018).

Theorem 1 (Carbonell-Nicolau and Llavador (2018, Theorem 1))**.** *Given $u \in \mathscr{U}^*$, a tax schedule in $\mathscr{T}$ is u-iir only if it is marginal-rate progressive.*

Thus, marginal-rate progressivity is necessary for a tax schedule to be inequality reducing. With endogenous income (and unlike in the endowment economy framework of Jakobsson (1976) and Fellman (1976)), the effect of a tax on gross incomes, in addition to its shape, determines the distributional effects of taxation. This suggests that consumer preferences, and their interaction with tax structures, are bound to play an important role in the formulation of inequality reducing properties of tax systems. The main result in Carbonell-Nicolau and Llavador (2018) demonstrates that this is indeed the case: only certain classes of preferences guarantee that the set of all marginal-rate progressive taxes are *iir*.

The contribution of this paper is twofold. First, we show that requiring *all* marginal-rate progressive tax schedules to be *iir* may be overly restrictive. Indeed, such a requirement rules out common classes of preferences, such as the CES and the quasi-linear utility functions. Second, we extend the analysis in Carbonell-Nicolau and Llavador (2018) by identifying necessary and sufficient conditions on preferences for various *subsets* of progressive tax schedules to be *iir*. This allows us to characterize the preferences in the CES and quasi-linear families and their corresponding subsets of *iir* tax schedules (Section 4).

Consider subclasses of $\mathscr{T}_{prog}$ characterized by the number of brackets and the ranges for their intercepts with the vertical axis and their marginal tax rates. Formally, given $K \in \mathbb{Z}_+$, $B \subseteq \mathbb{R}_+$ and subsets $R_0,\ldots,R_K$ of $[0,1)$, let $\mathscr{T}_{prog}(B,R_0,\ldots,R_K)$ be the set of all $(K+1)$-bracket marginal-rate progressive tax schedules $(\alpha_0,(t_0,\ldots,t_K),(\overline{y}_0,\ldots,\overline{y}_K)) \in \mathscr{T}_{prog}$ with intercept $\alpha_0 \in B$, marginal tax rates $t_0,\ldots,t_K$ with $t_k \in R_k$ for each $k \in \{0,\ldots,K\}$, and bracket thresholds $\overline{y}_1,\ldots,\overline{y}_K$, i.e.,

$$\mathscr{T}_{prog}(B,R_0,\ldots,R_K) := \left\{(\alpha_0,(t_0,\ldots,t_K),(\overline{y}_0,\ldots,\overline{y}_K)) \in \mathscr{T}_{prog} : \alpha_0 \in B \text{ and } (t_0,\ldots,t_K) \in R_0 \times \cdots \times R_K\right\}.$$

Let $\mathfrak{D}$ be the set of all $(B,(R_k)_{k=0}^{\infty})$ with $B \subseteq \mathbb{R}_+$ and $R_k \subseteq [0,1)$ for each k. For each $(B,(R_k)) \in \mathfrak{D}$, define

$$\mathscr{T}_{prog}(B,(R_k)) := \bigcup_{K \in \mathbb{Z}_+} \mathscr{T}_{prog}(B,R_0,\ldots,R_K).$$

When $R_0 = R_1 = \cdots = R$, we write $\mathscr{T}_{prog}(B,R)$ for $\mathscr{T}_{prog}(B,(R_k))$. Observe that $\mathscr{T}_{prog} = \mathscr{T}_{prog}(\mathbb{R}_+,[0,1))$.

Given $B \subseteq \mathbb{R}_+$ and $R \subseteq [0,1)$, define

$$\mathscr{T}_{lin}(B,R) := \{-b + ry \in \mathscr{T}_{lin} : b \in B \text{ and } r \in R\}$$

and

$$\overline{B} := \bigcup_{b\in B} \{b' \in \mathbb{R}_+ : b' \geq b\}.$$

The next theorem provides a necessary and sufficient condition for all the members in the subclass $\mathcal{T}_{prog}(\overline{B},(R_k))$ of $\mathcal{T}_{prog}$ to be *iir*.[4]

Theorem 2. *Given $u \in \mathcal{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$, $\mathcal{T}_{prog}(\overline{B},(R_k)) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if the members of $\mathcal{T}_{lin}(\overline{B},\bigcup_k R_k)$ are u-iir.*

Theorem 2 asserts that the members of the set $\mathcal{T}_{prog}(\overline{B},(R_k))$ of progressive tax schedules in $\mathcal{T}_{prog}$ whose intercept α_0 is greater than or equal to the infimum of $\overline{B}$ ($\inf\overline{B}$) and whose k-th marginal tax rate t_k lies in R_k are all *iir* if and only if all the linear taxes with intercept greater than or equal to $\inf\overline{B}$ and marginal tax rates in $\bigcup_k R_k$ are *iir*.

For $B = \mathbb{R}_+$ and $R_k = [0,1)$ for each k, one has that $\mathcal{T}_{prog}(B,(R_k)) = \mathcal{T}_{prog}$ and $\mathcal{T}_{lin}(\overline{B},\bigcup_k R_k) = \mathcal{T}_{lin}$; in this case, Theorem 2 immediately gives Theorem 2 in Carbonell-Nicolau and Llavador (2018):

Corollary 1 (to Theorem 2)**.** *Given $u \in \mathcal{U}^*$, $\mathcal{T}_{u\text{-}iir} = \mathcal{T}_{prog}$ if and only if the members of $\mathcal{T}_{lin}$ are u-iir.*

Theorem 2 implies that in order to determine whether the members of $\mathcal{T}_{prog}(\overline{B},(R_k))$ are *iir*, one can restrict attention to the inequality reducing properties of the subclass $\mathcal{T}_{lin}(\overline{B},\bigcup_k R_k)$ of linear tax schedules.

We now provide necessary and sufficient conditions on preferences under which the subclasses $\mathcal{T}_{prog}(\overline{B},(R_k))$ of marginal-rate progressive taxes are inequality reducing. In light of Theorem 2, this is tantamount to characterizing the family of preferences for which the members of a subset $\mathcal{T}_{lin}(B,R)$ are inequality reducing (this is done in Theorem 3 below). This characterization then allows us to present a variant of Theorem 2 in terms of first principles (see Theorem 4 below).

When T is a linear tax schedule in $\mathcal{T}_{lin}$ with $T(y) = -b$, where $b \geq 0$, we write $l^u(a,b)$ for $l^u(a,T)$. For each $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$, $l^u(a,b)$ is a solution to the problem

$$\max_{l\in[0,1]} u(al+b,l). \tag{3}$$

Since u is strictly quasiconcave on $\mathbb{R}_{++} \times [0,1)$, for each $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$, there is a unique solution $l^u(a,b)$ to (3). For given $b \geq 0$, the derivative of the map $a \mapsto l^u(a,b)$ exists for all but perhaps one $a > 0$.[5]

For $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$, define the *elasticity of income with respect to ability* at ability level a and endowment b as

$$\zeta^u(a,b) := \frac{\partial(al^u(a,b)+b)}{\partial a} \cdot \frac{a}{al^u(a,b)+b}.$$

[4]The theorem generalizes Theorem 2 in Carbonell-Nicolau and Llavador (2018) and can be proven using an adaptation of the proof of that theorem. The details are provided in the Appendix.

[5]This is proved in Carbonell-Nicolau and Llavador (2018, page 45).

Given $B \subseteq \mathbb{R}_+$ and $R \subseteq [0,1)$, let $\mathscr{U}(B,R)$ be the set of all $u \in \mathscr{U}^*$ satisfying the following condition:

$$\zeta^u((1-r)a,b) \leq \zeta^u(a,0), \quad \text{for all } (a,b,r) \in \mathbb{R}_{++} \times B \times R. \tag{4}$$

The following result shows that (4) is indeed the relevant condition to characterize the familiy of preferences for which the corresponding subclass of linear tax schedules, $\mathscr{T}_{lin}(B,R)$, is inequality reducing. The proof is relegated to the Appendix.[6]

Theorem 3. *For $u \in \mathscr{U}^*$, the members of $\mathscr{T}_{lin}(B,R)$ are u-iir if and only if $u \in \mathscr{U}(B,R)$.*

Now combining Theorem 2 and Theorem 3 yields the main result of this paper.[7]

Theorem 4. *Given $u \in \mathscr{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$, $\mathscr{T}_{prog}(\overline{B},(R_k)) \subseteq \mathscr{T}_{u\text{-}iir} \subseteq \mathscr{T}_{prog}$ if and only if $u \in \mathscr{U}(\overline{B},\bigcup_k R_k)$.*

Theorem 4 asserts that the members of the set $\mathscr{T}_{prog}(\overline{B},(R_k))$ of progressive tax schedules in $\mathscr{T}_{prog}$ whose intercept α_0 is greater than or equal to $\inf \overline{B}$ and whose k-th marginal tax rate t_k lies in R_k are all iir if and only if $u \in \mathscr{U}(\overline{B},\bigcup_k R_k)$, i.e., if and only if the elasticity of income with respect to ability satisfies condition (4). The next section evaluates this elasticity condition within two important families of preferences—the CES and the quasi-linear preferences—and illustrates that this condition is intimately related to the elasticity of substitution between consumption and leisure.

4 Applications

This section characterizes the subclasses of progressive taxes that are inequality reducing for two commonly used families of income-leisure preferences: the CES and the quasi-linear preferences. The CES utility function (often in its Cobb-Douglas version) is very common in the literature on life-cycle models (Heckman and MaCurdy, 1982; French, 2005; Blundell et al., 2016), while static models with fixed costs traditionally work with quasi-linear preferences (Cogan, 1981).[8] These utilities are also dominant in surveys and textbooks on labor supply and fiscal policy (Pencavel, 1986; Killingsworth and Heckman, 1986; Auerbach and Kotlikoff, 1987; Keane, 2011; Blundell et al., 2016).

We find that the combination of a sufficiently high subsidy and a sufficiently large consumption-leisure elasticity of substitution suffice for marginal-rate progressive tax schedules to be income-inequality reducing.

For each case, we first specify the family of utility functions and calculate their elasticities. We then characterize, as an application of Theorem 4, the utility parameters for which there

[6]Theorem 3 subsumes Theorem 3 in Carbonell-Nicolau and Llavador (2018), which states that the members of $\mathscr{T}_{lin}$ are u-iir if and only if $u \in \widehat{\mathscr{U}}$, where $\widehat{\mathscr{U}}$ is the class of utility functions $u \in \mathscr{U}^*$ satisfying the following two conditions: (*i*) $\zeta^u(a,b) \leq \zeta^u(a,0)$ for all $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$; and (*ii*) the map $a \mapsto \zeta^u(a,0)$ defined on $\mathbb{R}_{++}$ is nondecreasing. It follows from the proof of Theorem 3 in Carbonell-Nicolau and Llavador (2018) that $\widehat{\mathscr{U}} = \mathscr{U}(\mathbb{R}_+,[0,1))$.

[7]This result refines Corollary 3 in Carbonell-Nicolau and Llavador (2018).

[8]Static models tend to specify a labor supply function directly, which makes it difficult to identify a widely used utility function (Keane, 2011, page 966).

exist classes of iir tax schedules and develop intuition for our findings. Formal proofs are relegated to the Appendix.

4.1 CES utility

Consider the well-known CES utility function

$$u(c,l) := \begin{cases} c^{\gamma} + \beta(1-l)^{\gamma} & \text{if } \gamma \in (0,1), \\ -c^{\gamma} - \beta(1-l)^{\gamma} & \text{if } \gamma < 0, \end{cases} \tag{5}$$

where the parameter γ determines the elasticity of substitution between consumption and leisure, and β is a positive constant. One has

$$l^u(a,b) = \begin{cases} \dfrac{\left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} - b}{a + \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}}} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ 0 & \text{otherwise,} \end{cases}$$

$$al^u(a,b) + b = \begin{cases} \dfrac{\left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}}(a+b)}{a + \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}}} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ b & \text{otherwise,} \end{cases}$$

$$\zeta^u(a,0) = \begin{cases} \dfrac{(1-\gamma)a^{\frac{2}{1-\gamma}} + a^{\frac{2-\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}}{(1-\gamma)a^{\frac{2}{1-\gamma}} + (1-\gamma)a^{\frac{2-\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\zeta^u((1-r)a,b) = \begin{cases} \dfrac{(1-\gamma)((1-r)a)^{\frac{2}{1-\gamma}} + \beta^{\frac{1}{1-\gamma}} b\gamma((1-r)a)^{\frac{1}{1-\gamma}} + \beta^{\frac{1}{1-\gamma}}((1-r)a)^{\frac{2-\gamma}{1-\gamma}}}{(1-\gamma)((1-r)a)^{\frac{\gamma}{1-\gamma}}((1-r)a+b)[(1-r)a\beta^{\frac{1}{1-\gamma}} + ((1-r)a)^{\frac{1}{1-\gamma}}]} & \text{if } \left(\frac{(1-r)a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 1. *Let u be the CES utility function given in (5). Suppose that $R \subseteq [0,1)$ and $\sup R < 1$. Then there exists $\underline{b} \geq 0$ such that $\mathcal{T}_{prog}(B^*, R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $\gamma \in [\frac{1}{2}, 1)$, where $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$.*

As γ tends to 1, the CES utility function becomes linear and consumption and leisure become perfect substitutes. As γ tends to $-\infty$, the indifference curves become “right angles,” *i.e.*, the utility function regards the two goods as perfect complements.

Proposition 1 states that when the elasticity of substitution is large enough, *i.e.*, when consumption and leisure substitute “sufficiently well” for each other, there are (nonempty) subclasses of progressive tax schedules whose members are inequality reducing. Specifically, in this case it suffices to choose a sufficiently large subsidy for a progressive tax schedule to be inequality reducing.

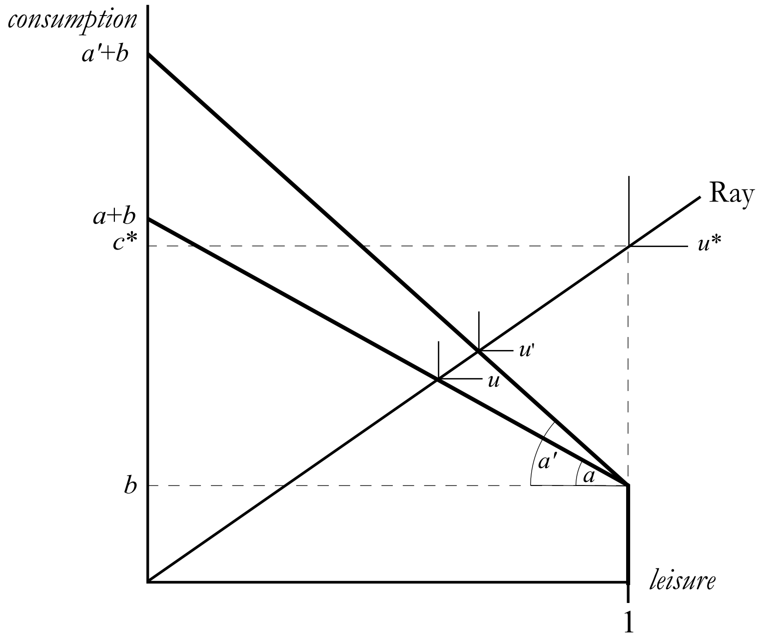

Figure 1: **Perfect complements**. Individual choice for different ability levels and exogenous income b. The ray represents the bundles with the "correct" proportions between leisure and consumption. The maximum utility level u^* is attained for the bundle $(1, c^*)$. For sufficiently high abilities, income (the chosen level of consumption) cannot increase at increasing rates. Hence, the elasticity of income to ability must decrease and not all progressive taxes are income inequality reducing (Theorem 3 and Corollary 1).

In order to capture the intuition behind this result, recall that a necessary and sufficient condition for the containments $\mathscr{T}_{prog}(B^*,R) \subseteq \mathscr{T}_{u-iir} \subseteq \mathscr{T}_{prog}$ to hold is that $u \in \mathscr{U}(B^*,R)$, and consider first the opposite extreme case when consumption and leisure are perfect complements. Because leisure is bounded above by 1, utility has an upper bound, with $(1, c^*)$ as the optimal bundle, where c^* is the "ideal" consumption level corresponding to maximum leisure (see Figure 1). Individuals of higher ability, whose consumption entails a lower opportunity cost in terms of leisure time, choose higher consumption levels. As a grows large, the optimal consumption level converges to c^*. Hence, for a sufficiently large ability, the change in consumption (and hence in income) with respect to ability increases less than proportionally in ability. This implies that the elasticity of income with respect to ability must decrease at some point, and so $u \notin \mathscr{U}(B^*,R)$.

At the other extreme, when consumption and leisure are treated as perfect substitutes, higher ability individuals will choose zero leisure and low ability individuals zero labor, implying that the elasticity of income with respect to ability is not decreasing (Figure 2).

For non-extreme cases, when consumption and leisure are not good substitutes, one can always find individuals of sufficiently high ability for whom the elasticity of income with respect to ability decreases, as in the case of perfect complements. Alternatively, when consumption and leisure substitute relatively well, the elasticity condition is satisfied for sufficiently high ability levels and can also be guaranteed for low ability agents by equalizing

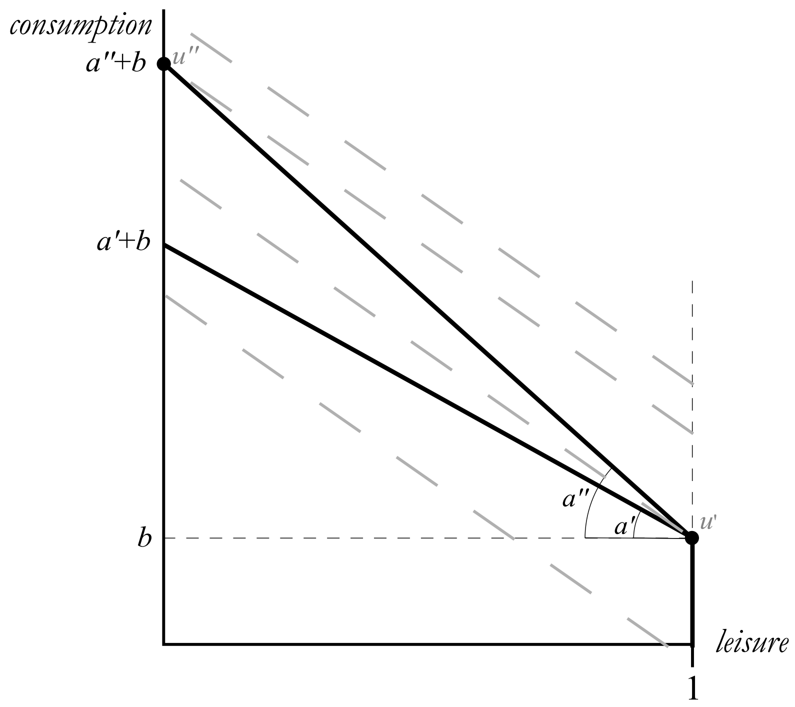

Figure 2: **Perfect substitutes**. Individual choice for different ability levels and exogenous income b. Individuals with a sufficiently high ability, like a'', choose zero leisure; while those with sufficiently low ability, like a', choose zero labor. Hence, the elasticity of income to ability is non-decreasing and all progressive taxes are income inequality reducing (Theorem 3 and Corollary 1).

their incomes with a sufficiently large subsidy. Hence the condition B^* in the statement of Proposition 1.

Remark 1. When $\gamma \to 0$ the CES utility function converges to the Cobb-Douglas utility function, and in this limiting case (15) holds for all $(a,b,r) \in \mathbb{R}_{++} \times [0,+\infty) \times [0,1)$, and so Theorem 4 implies that marginal-rate progressive and only marginal-rate progressive tax schedules are *iir*, i.e., $\mathcal{T}_{u\text{-}iir} = \mathcal{T}_{prog}$.[9]

4.2 Quasi-linear utility

Consider the quasi-linear utility function

$$u(c,l) := c + \frac{\beta(1-l)^{1-\delta}}{1-\delta}, \tag{6}$$

where $\beta > 0$ and $\delta > 0$, with $\delta \neq 1$.[10] One has

$$l^u(a,b) = \begin{cases} 1 - \left(\frac{a}{\beta}\right)^{-1/\delta} & \text{if } a \geq \beta \\ 0 & \text{if } a \leq \beta, \end{cases}$$

[9]This was established in Carbonell-Nicolau and Llavador (2018, Remark 3).
[10]The $MRS(c,l)$ tends to $+\infty$ as $l \to 1^-$ (recall the Inada condition in (1)) if and only if $\delta > 0$.

and

$$al^u(a,b)+b = \begin{cases} a+b-a\left(\frac{a}{\beta}\right)^{-1/\delta} & \text{if } a \geq \beta \\ b & \text{if } a \leq \beta. \end{cases}$$

Define $\theta(a) := \left(\frac{a}{\beta}\right)^{-1/\delta}$ and $\theta_r(a) := \left(\frac{(1-r)a}{\beta}\right)^{-1/\delta}$. Note that $\theta(a) > 1$ for $a \geq \beta$, and $\theta_r(a) > 1$ for $a \geq \frac{\beta}{1-r} > \beta$. Compute

$$\zeta^u(a,0) = \begin{cases} 1 - \frac{1}{\delta(1-\theta(a))} & \text{if } a \geq \beta, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\zeta^u((1-r)a,b) = \begin{cases} \frac{a(1-r)-(1-\theta_r(a))(1-r)a\delta}{b\theta_r(a)\delta-(1-\theta_r(a))(1-r)a\delta} & \text{if } a \geq \frac{\beta}{(1-r)}, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 2. *Let u be the quasi-linear utility function given in* (6)*. Suppose that $R \subseteq [0,1)$ and* $\sup R < 1$*. Then there exists $\underline{b} \geq 0$ such that $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $\delta \in (0,1)$, where $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$.*

For the quasi-linear utility function, the parameter δ determines the inverse of the elasticity of leisure with respect to ability, so that leisure is elastic for $\delta < 1$ and inelastic for $\delta > 1$.[11] Proposition 2 states that when the demand of leisure is inelastic, any progressive tax schedule with a sufficiently large subsidy is inequality reducing.

The intuition for this result is similar to that for the CES utility function. Observe that consumption and leisure become perfect substitutes for $\delta = 0$, and that the degree of substitutability decreases as δ increases.[12] As $\delta \to 0$, the demand of leisure becomes perfectly elastic, with consumption and leisure becoming perfect substitutes. High ability individuals choose zero leisure and low ability individuals zero labor, implying that the elasticity of income with respect to ability is not decreasing. (Recall Figure 2.)

For $\delta < 1$, leisure demand is elastic and an ability increase leads to a more than proportional leisure decrease, producing a larger increase in income for sufficiently large ability levels, and satisfying the elasticity condition for sufficiently large ability levels. The elasticity condition can also be guaranteed for low ability individuals by equalizing their incomes with a sufficiently large subsidy. Hence the condition B^* in the statement of Proposition 1.

5 Concluding remarks

This paper characterizes consumer preferences for which various subclasses of progressive tax schedules are inequality reducing. The framework considered here, which subsumes that in Carbonell-Nicolau and Llavador (2018), allows one to expand the set of consumer preferences by suitably restricting the set of progressive taxes. This is illustrated in Section 4 for two

[11] It is easy to obtain the demand of leisure as $(a/\beta)^{-1/\delta}$ for $a \geq \beta$, and hence the elasticity of leisure to ability as $-1/\delta$.

[12] Both the quasi-linear utility function considered here and the CES utility function from Subsection 4.1 become the linear utility function $u(c,l) = c + \beta(1-l)$ for $\delta = 0$ and $\gamma = 1$, respectively.

standard families of utility functions: the CES and the quasi-linear utility functions. Indeed, the characterizations in Proposition 1 and Proposition 2 state that it suffices to choose a sufficiently large subsidy for a progressive tax schedule to be inequality reducing if and only if the elasticity of substitution between consumption and leisure is large enough. In this regard, while the relevant parameter regions may differ across families of consumer preferences, our discussions in Section 4 suggest that the elasticity of substitution between consumption and leisure determines, in a crucial manner, the existence of nonempty subclasses of inequality reducing tax systems.

Appendix

In this appendix we present the proofs of Theorem 2, Theorem 3, Proposition 1, and Proposition 2. Each proof is preceded by a restatement of its corresponding theorem for the convenience of the reader.

The proofs of Theorem 2 and that of Theorem 3 adapt arguments from the proofs of Theorem 2 and Theorem 3 in Carbonell-Nicolau and Llavador (2018).

The following two lemmas, whose proofs can be found in Carbonell-Nicolau and Llavador (2018) (see their Lemma 2 and Lemma 3), are instrumental in the proofs of Theorem 2 and Theorem 3.

Lemma 1. *Given $u \in \mathscr{U}$, $(c,y) \in \mathbb{R}^2_{++}$, and $q \in (0,+\infty)$, there exists an $a > y$ such that $\eta^a(c,y) = q$.*

Lemma 2. *Given $u \in \mathscr{U}^*$, a tax schedule $T \in \mathscr{T}$ is u-iir if and only if for any ability distribution $\boldsymbol{a} \in \mathscr{A}$ and for any pre-tax and post-tax income functions y^u and x^u,*

$$\frac{x^u(a_i,T)}{y^u(a_i,0)} \geq \frac{x^u(a_{i+1},T)}{y^u(a_{i+1},0)} \qquad \forall i \in \{1,\dots,n-1\} : y^u(a_i,0) > 0. \tag{7}$$

A Proof of Theorem 2

Theorem 2. *Given $u \in \mathscr{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$, $\mathscr{T}_{prog}(\overline{B},(R_k)) \subseteq \mathscr{T}_{u\text{-}iir} \subseteq \mathscr{T}_{prog}$ if and only if the members of $\mathscr{T}_{lin}(\overline{B},\bigcup_k R_k)$ are u-iir.*

Proof. Suppose that $u \in \mathscr{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$.

Since $\mathscr{T}_{lin}(\overline{B},\bigcup_k R_k) \subseteq \mathscr{T}_{prog}(\overline{B},(R_k))$, the 'only if' part of the equivalence is obvious.

Assume now that the members of $\mathscr{T}_{lin}(\overline{B},\bigcup_k R_k) \subseteq \mathscr{T}_{u\text{-}iir}$. We need to prove that $\mathscr{T}_{prog}(\overline{B},(R_k)) \subseteq \mathscr{T}_{u\text{-}iir}$. By Lemma 2, this is equivalent to showing that that condition (7) holds for any $T \in \mathscr{T}_{prog}(\overline{B},(R_k))$, for any ability distribution $\boldsymbol{a} \in \mathscr{A}$, and for any pre-tax and post-tax income functions y^u and x^u.

Take $T = (\alpha_0,\boldsymbol{t},\overline{\boldsymbol{y}}) \in \mathscr{T}_{prog}(\overline{B},(R_k))$ and, for each income threshold $\overline{y}_k$ of T, define the linear tax schedule $T_k(y) := t_k y - \alpha_k$ with $\alpha_0 \in \overline{B}$, $t_k \in R_k$ for $k \in \{0,...,K\}$, and $\alpha_k := \alpha_{k-1} + (t_k - t_{k-1})\overline{y}_k$ for $k \in \{1,\dots,K\}$.

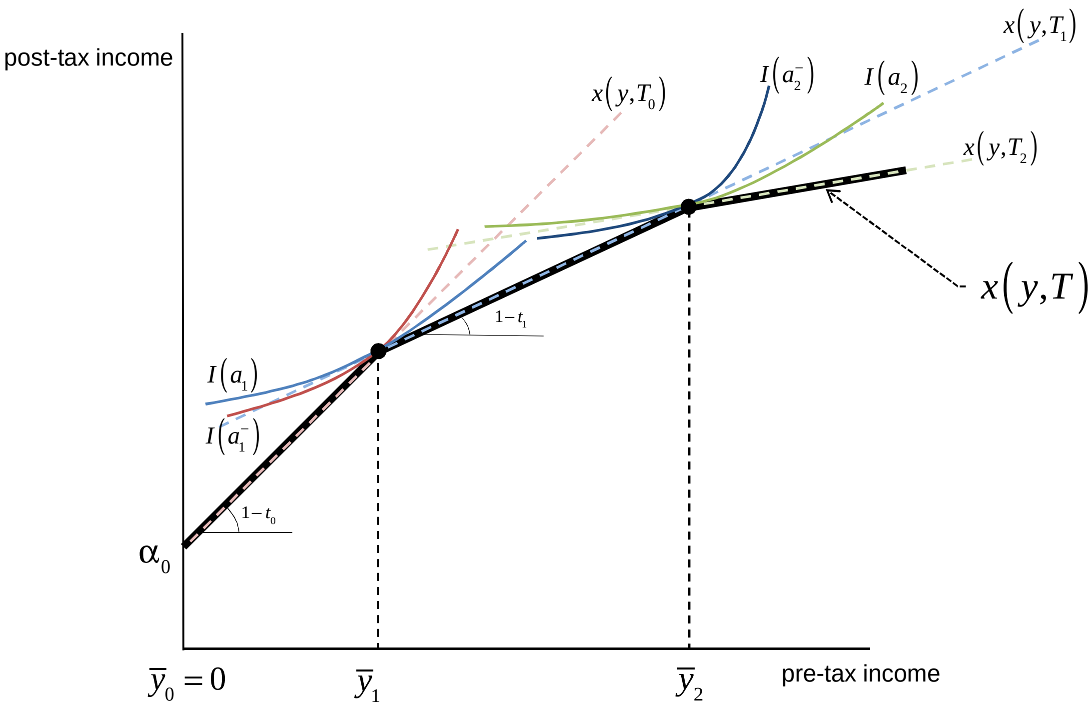

Figure 3: Figure for Theorem 2

Pre-tax and post-tax income functions, y^u and x^u, are uniquely defined, since preferences are strictly quasiconcave and the tax function T is convex. For $k \in \{1,...,K\}$, define the abilities a_k^- and a_k such that

$$a_k^- := \min\{a : y^u(a, T_{k-1}) = \bar{y}_k^u\} \quad \text{and} \quad a_k := \max\{a : y(a, T_k) = \bar{y}_k\}$$

(see Figure 3). Lemma 1 guarantees that a_k^- and a_k exist and are well-defined for all $k \in \{1,\ldots,K\}$.

Furthermore, since T is marginal-rate progressive (and hence $t_{k-1} < t_k$ for all $k \in \{1,\ldots,K\}$), agent monotonicity (Definition 4) implies that $a_k^- \le a_k < a_{k+1}^-$.

Next, define the following family of sets covering $(0,+\infty)$:

$$\mathfrak{A} := \left\{(0, a_1^-], \{[a_k^-, a_k]\}_{k=1}^{K}, \{[a_k, a_{k+1}^-]\}_{k=1}^{K-1}, [a_K, +\infty)\right\}.$$

We first show that condition (7) is satisfied for ability distributions contained in each element of the family $\mathfrak{A}$.

(*i*) Consider first the interval $(0, a_1^-]$. Observe that $y^u(a,T) = y^u(a,T_0)$ for all $a \le a_1^-$. Because T_0 is a linear tax, it is *u-iir*, and so Lemma 2 gives

$$\frac{x^u(a,T)}{y^u(a,0)} = \frac{x^u(a,T_0)}{y^u(a,0)} \ge \frac{x^u(a',T_0)}{y^u(a',0)} = \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a \le a' \le a_1^-. \tag{8}$$

(*ii*) For $[a_K,+\infty)$, a symmetric argument shows that

$$\frac{x^u(a,T)}{y^u(a,0)} \geq \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a_K \leq a \leq a'. \tag{9}$$

(*iii*) Now consider the interval $[a_k^-, a_k]$ for $k \in \{1,\dots,K\}$. Observe that

$$y^u(a_k,T) = y^u(a_k,T_k) = \overline{y}_k = y^u(a_k^-,T_{k-1}) = y^u(a_k^-,T).$$

Because the map $a \mapsto y^u(a,T)$ is monotone (Mirrlees, 1971, Theorem 1), $y^u(a,T) = \overline{y}_k$ for all $a \in [a_k^-,a_k]$, and $y^u(a',0) \geq y^u(a,0)$ for all $a_k^- \leq a \leq a' \leq a_k$. Therefore,

$$\frac{x^u(a,T)}{y^u(a,0)} = \frac{\overline{y}_k - T(\overline{y}_k)}{y^u(a,0)} \geq \frac{\overline{y}_k - T(\overline{y}_k)}{y^u(a',0)} = \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a,a' \in [a_k^-,a_k],\ a \leq a'. \tag{10}$$

(*iv*) Finally, consider the interval $[a_k, a_{k+1}^-]$ for $k \in \{1,\dots,K-1\}$. By construction, we have $y^u(a,T) = y^u(a,T_k)$ for all $a \in [a_k, a_{k+1}^-]$. Therefore, since T_k is a linear tax in $\mathcal{T}_{lin}((\overline{B}, \bigcup_k R_k)$, and hence *u-iir*, Lemma 2 gives

$$\frac{x^u(a,T)}{y^u(a,0)} = \frac{x^u(a,T_k)}{y^u(a,0)} \geq \frac{x^u(a',T_k)}{y^u(a',0)} = \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a,a' \in [a_k,a_{k+1}^-],\ a \leq a'. \tag{11}$$

Combining equations (8)-(11) we obtain (7) for every $\boldsymbol{a} \in \mathcal{A}$. ■

B Proof of Theorem 3

Theorem 3. *For $u \in \mathcal{U}^*$, the members of $\mathcal{T}_{lin}(B,R)$ are u-iir if and only if $u \in \mathcal{U}(B,R)$.*

Proof. Given $u \in \mathcal{U}^*$, $B \subseteq \mathbb{R}_+$, and $R \subseteq [0,1)$, the members $T(y) = -b + ry$ of $\mathcal{T}_{lin}(B,R)$ are u-iir if and only if the map

$$a \mapsto \frac{x^u(a,T)}{y^u(a,0)} = \frac{a(1-r)l^u(a,T)+b}{al^u(a,0)} = \frac{a(1-r)l^u((1-r)a,b)+b}{al^u(a,0)} \tag{12}$$

defined on $\mathbb{R}_{++}$ is nonincreasing for every $(b,r) \in B \times R$ (Lemma 2). Equivalently, the members of $\mathcal{T}_{lin}(B,R)$ are u-iir if and only if

$$\begin{aligned} &\frac{(1-r)\left((1-r)a'\frac{\partial l^u((1-r)a',b)}{\partial a} + l^u((1-r)a',b)\right)a'l^u(a',0)}{(a'l^u(a',0))^2} \\ &\qquad - \frac{((1-r)a'l^u((1-r)a',b)+b)\left(a'\frac{\partial l^u(a',0)}{\partial a} + l^u(a',0)\right)}{(a'l^u(a',0))^2} \leq 0 \end{aligned} \tag{13}$$

for every $(a',b,r) \in \mathbb{R}_{++} \times B \times R$.[13] Since the above inequality can be expressed as

$$\frac{(1-r)a'\left((1-r)a'\frac{\partial l^u((1-r)a',b)}{\partial a} + l^u((1-r)a',b)\right)}{(1-r)a'l^u((1-r)a',b)+b} \le \frac{a'\left(a'\frac{\partial l^u(a',0)}{\partial a} + l^u(a',0)\right)}{a'l^u(a',0)},$$

or, equivalently, as

$$\zeta^u((1-r)a',b) \le \zeta^u(a',0), \tag{14}$$

we see that the members of $\mathcal{T}_{lin}(B,R)$ are u-iir if and only if (14) holds for every $(a',b,r) \in \mathbb{R}_{++} \times B \times R$. Consequently, for $u \in \mathcal{U}^*$, the members of $\mathcal{T}_{lin}(B,R)$ are u-iir if and only if $u \in \mathcal{U}(B,R)$. ■

C Proof of Proposition 1

Proposition 1. *Let u be the CES utility function given in (5). Suppose that $R \subseteq [0,1)$ and $\sup R < 1$. Then there exists $\underline{b} \ge 0$ such that $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $\gamma \in [\frac{1}{2},1)$, where $B^* := \{b \in \mathbb{R}_+ : b \ge \underline{b}\}$.*

Proof. Since $u \in \mathcal{U}^*$, given $\underline{b} \ge 0$, $B^* := \{b \in \mathbb{R}_+ : b \ge \underline{b}\}$, and $R \subseteq [0,1)$, Theorem 4 gives $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $u \in \mathcal{U}(B^*,R)$.

Given $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$, if $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} < b$, then $(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} < b$, implying $\zeta^u(a,0) = 0 \ge 0 = \zeta^u((1-r)a,b)$. If $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \ge b > (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}}$, then $\zeta^u(a,0) \ge 0 = \zeta^u((1-r)a,b)$. If $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \ge (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \ge b$, then $\zeta^u(a,0) \ge \zeta^u((1-r)a,b)$ if and only if

$$\frac{(1-\gamma)a^{\frac{2}{1-\gamma}} + a^{\frac{2-\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}}{(1-\gamma)a^{\frac{2}{1-\gamma}} + (1-\gamma)a^{\frac{2-\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}} \ge \frac{(1-\gamma)((1-r)a)^{\frac{2}{1-\gamma}} + \beta^{\frac{1}{1-\gamma}}b\gamma((1-r)a)^{\frac{1}{1-\gamma}} + \beta^{\frac{1}{1-\gamma}}((1-r)a)^{\frac{2-\gamma}{1-\gamma}}}{(1-\gamma)((1-r)a)^{\frac{\gamma}{1-\gamma}}((1-r)a+b)[(1-r)a\beta^{\frac{1}{1-\gamma}} + ((1-r)a)^{\frac{1}{1-\gamma}}]}.$$

Arranging terms gives

$$\begin{aligned} a^{\frac{1}{1-\gamma}}(a(1-r))^{\frac{1}{1-\gamma}}\Big[&(a(1-r))^{-1}((1-\gamma)a^{\frac{1}{1-\gamma}} + a\beta^{\frac{1}{1-\gamma}})(a(1-r)\beta^{\frac{1}{1-\gamma}} + (a(1-r))^{\frac{1}{1-\gamma}}) \\ &-\gamma\beta^{\frac{1}{1-\gamma}}(a^{\frac{1}{1-\gamma}} + a\beta^{\frac{1}{1-\gamma}})\Big]b \\ \ge a^{\frac{1}{1-\gamma}}(a(1-r))^{\frac{1}{1-\gamma}}\Big[&(a^{\frac{1}{1-\gamma}} + a\beta^{\frac{1}{1-\gamma}})(a(1-r)\beta^{\frac{1}{1-\gamma}} + (1-\gamma)(a(1-r))^{\frac{1}{1-\gamma}}) \\ &-((1-\gamma)a^{\frac{1}{1-\gamma}} + a\beta^{\frac{1}{1-\gamma}})(a(1-r)\beta^{\frac{1}{1-\gamma}} + (a(1-r))^{\frac{1}{1-\gamma}})\Big]. \end{aligned}$$

This simplifies to

$$\begin{aligned} \Big[((1-\gamma)a^{\frac{1}{1-\gamma}} + a\beta^{\frac{1}{1-\gamma}})(\beta^{\frac{1}{1-\gamma}} + (a(1-r))^{\frac{\gamma}{1-\gamma}}) - \gamma\beta^{\frac{1}{1-\gamma}}(a^{\frac{1}{1-\gamma}} + a\beta^{\frac{1}{1-\gamma}})\Big]b \\ \ge a^{1+\frac{1}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\gamma(1-r-(1-r)^{\frac{1}{1-\gamma}}). \end{aligned} \tag{15}$$

[13] More precisely, the map defined in (12) is nonincreasing for every $(b,r) \in B \times R$ if and only if for every $(b,r) \in B \times R$, (13) holds for all but perhaps one $a' > 0$.

We claim that if $\sup R < 1$ there exists $\underline{b} \geq 0$ such that $\mathscr{T}_{prog}(B^*,R) \subseteq \mathscr{T}_{u\text{-}iir} \subseteq \mathscr{T}_{prog}$ if and only if $\gamma \in [\frac{1}{2},1)$. To see this, it suffices to show that (i) for $\gamma < \frac{1}{2}$ and $\underline{b} \geq 0$, there exists $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \geq (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$ such that (15) does not hold, and (ii) for $\gamma \in [\frac{1}{2},1)$, there exists $\underline{b} \geq 0$ such that for $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \geq (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$, (15) holds.

Given $\underline{b} \geq 0$ and $\gamma < 0$, the bracketed term on the left-hand side of (15) is positive and the right-hand side of (15) divided by the bracketed term on the left-hand side of (15) converges to infinity as a tends to infinity. Consequently, there exists $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \geq (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$ such that (15) does not hold. If $0 < \gamma < \frac{1}{2}$, the right-hand side of (15) is positive, the bracketed term on the left-hand side of (15) is positive for a large enough, and the right-hand side of (15) divided by the bracketed term on the left-hand side of (15) converges to infinity as a tends to infinity. Therefore, there exists $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \geq (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$ such that (15) does not hold.

If $\gamma = \frac{1}{2}$, the bracketed term on the left-hand side of (15) is positive and the and the right-hand side of (15) divided by the bracketed term on the left-hand side of (15) is increasing in a and converges to $r\beta^2$ as $a \to \infty$. Consequently, it suffices to set $\underline{b} = \beta^2 \sup R \leq \beta^2$.[14]

If $\gamma \in (\frac{1}{2},1)$, the bracketed term on the left-hand side of (15) is positive for large enough a, and the right-hand side of (15) divided by the bracketed term on the left-hand side of (15) tends to 0 as $a \to \infty$, and so if $\sup R < 1$ there exists $\underline{b}$ such that for $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$, (15) holds.[15] ■

D Proof of Proposition 2

Proposition 2. *Let u be the quasi-linear utility function given in* (6)*. Suppose that $R \subseteq [0,1)$ and $\sup R < 1$. Then there exists $\underline{b} \geq 0$ such that $\mathscr{T}_{prog}(B^*,R) \subseteq \mathscr{T}_{u\text{-}iir} \subseteq \mathscr{T}_{prog}$ if and only if $\delta \in (0,1)$, where $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$.*

Proof. Since $u \in \mathscr{U}^*$, given $\underline{b} \geq 0$, $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$, and $R \subseteq [0,1)$, Theorem 4 gives $\mathscr{T}_{prog}(B^*,R) \subseteq \mathscr{T}_{u\text{-}iir} \subseteq \mathscr{T}_{prog}$ if and only if $u \in \mathscr{U}(B^*,R)$.

[14] For example, if $\beta = 1$ and the maximum marginal tax rate is $\frac{1}{2}$, then it suffices to consider the set of all the marginal-rate progressive tax schedules that provide a subsidy of at least \$$\frac{1}{2}$ for those individuals with no income.

[15] For instance, if $\gamma = \frac{3}{4}$, $\sup R = 0.6$, and $\beta = 1$, (15) becomes

$$\left[(0.25a^4 + a)(1 + (a(1-r))^3) - 0.75(a^4 + a)\right] b \geq 0.75a^4(1-r)(1-(1-r)^3). \tag{16}$$

If $\underline{b} = 81$, then given $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} = (1-r)^4 a^4 \geq b \geq \underline{b}$, i.e., $a \geq \frac{b^{\frac{1}{4}}}{1-r} \geq \frac{\underline{b}^{\frac{1}{4}}}{1-r} = \frac{3}{1-r}$, (16) is equivalent to

$$b \geq \frac{0.75a^4(1-r)(1-(1-r)^3)}{(0.25a^4 + a)(1 + (a(1-r))^3) - 0.75(a^4 + a)}, \tag{17}$$

and since

$$\frac{0.75a^4(1-r)(1-(1-r)^3)}{(0.25a^4 + a)(1 + (a(1-r))^3) - 0.75(a^4 + a)} < 81$$

and $b \geq \underline{b}$, it follows that (17) holds.

Given $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$, if $a \le \beta/(1-r)$, then $\zeta^u((1-r)a,b) = 0 \le \zeta^u(a,0)$. If $a > \beta/(1-r)$, then $\zeta^u(a,0) \ge \zeta^u((1-r)a,b)$ if and only if

$$1 - \frac{1}{\delta(1-\theta(a))} \ge \frac{a(1-r) - (1-\theta_r(a))(1-r)a\delta}{b\theta_r(a)\delta - (1-\theta_r(a))(1-r)a\delta}.$$

Using $\theta(a) > \theta_r(a) > 1$, and arranging terms, we obtain

$$b\,(\delta(\theta(a)-1)\theta_r(a)\delta + \theta_r(a)\delta) \ge (\theta(a)-1)(1-r)a\delta - (\theta_r(a)-1)(1-r)a\delta.$$

This simplifies to

$$b \ge \frac{(\theta(a)-\theta_r(a))(1-r)a}{(1+\delta(\theta(a)-1))\theta_r(a)}.$$

Finally, since $\theta_r(a) = (1-r)^{1/\delta}\theta(a) = (1-r)^{1/\delta}\left(\frac{a}{\beta}\right)^{-1/\delta}$,

$$b \ge \left((1-r)^{-1/\delta} - 1\right)(1-r)\frac{a}{1+\left(\left(\frac{a}{\beta}\right)^{1/\delta} - 1\right)\delta} = K(r)\varphi(a), \tag{18}$$

where $K(r) := \left((1-r)^{-1/\delta} - 1\right)(1-r) > 0$ for all $r \in (0,1)$, and $\varphi(a) := \frac{a}{1+\left((a/\beta)^{1/\delta}-1\right)\delta}$ for $a \ge \beta$.

We claim that if $\sup R < 1$ there exists $\underline{b} \ge 0$ such that $\mathcal{T}_{prog}(B^*, R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $\delta \in (0,1)$. To see this, it suffices to show that (i) for $\delta > 1$ and $\underline{b} \ge 0$, there exists $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $a > \beta$ such that (18) does not hold; and (ii) for $\delta \in (0,1)$, there exists $\underline{b} \ge 0$ such that for $(a,b,r) \in [\beta,\infty) \times B^* \times R$, (18) holds.

Given $\underline{b} \ge 0$ and $\delta > 1$, $\varphi(a)$ converges to infinity as a tends to infinity, and consequently, there exists $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $a > \beta$ such that (18) does not hold.

If $\delta \in (0,1)$, $\varphi(a)$ tends to 0 as $a \to \infty$ and $K(r)$ is monotone increasing. Hence, if $\sup R < 1$, there exists $\underline{b}$ such that for $(a,b,r) \in [\beta,\infty) \times B^* \times R$, (18) holds. ■

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